Intro Video: Section 2.3 part 2
Calculating using limit laws

Math F251X: Calculus I

Example:

$$\lim_{x \to -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$$

Direct substitution?

Looks like $\dfrac{(-4)^2 + 5(-4) + 4}{(-4)^2 + 3(-4) - 4} = \dfrac{16 - 20 + 4}{16 - 12 - 4} = \dfrac{0}{0}$

No good! Need algebra!

$$= \lim_{x \to -4} \frac{(x+1)\cancel{(x+4)}}{(x-1)\cancel{(x+4)}}$$

$$= \lim_{x \to -4} \frac{x+1}{x-1}$$

$$= \frac{-4+1}{-4-1}$$

$$= \frac{-3}{-5} = \frac{3}{5}$$

Example:

$$\lim_{x \to 16} \frac{4-\sqrt{x}}{16x - x^2}$$

"type" $0/0$
if we direct substitute

$$= \lim_{x \to 16} \left(\frac{4-\sqrt{x}}{16x - x^2}\right)\left(\frac{4+\sqrt{x}}{4+\sqrt{x}}\right)$$

Algebraic trick!
"multiply by the conjugate"
$a - \sqrt{b}$ has conjugate $a + \sqrt{b}$
and vice versa.

$$= \lim_{x \to 16} \frac{16 - x}{(16x - x^2)(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{\cancel{16-x}}{x\,\cancel{(16-x)}(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{1}{x(4+\sqrt{x})}$$

$$= \frac{1}{16(4+\sqrt{16})} = \frac{1}{16(8)} = \frac{1}{128}$$

$$\begin{array}{r} {}^{4}16 \\ 8 \\ \hline 128 \end{array}$$

Example: $\lim\limits_{h \to 0} \dfrac{(\sqrt{9+h}) - 3}{h}$

$$= \lim_{h \to 0} \left(\frac{(\sqrt{9+h}) - 3}{h}\right)\left(\frac{(\sqrt{9+h}) + 3}{(\sqrt{9+h}) + 3}\right)$$

$$= \lim_{h \to 0} \frac{(9+h) - 9}{h((\sqrt{9+h}) + 3)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{9+h} + 3}$$

$$= \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{6}$$

Example:

$$\lim_{x \to 2} \frac{xe^x - 2e^x}{x-2}$$

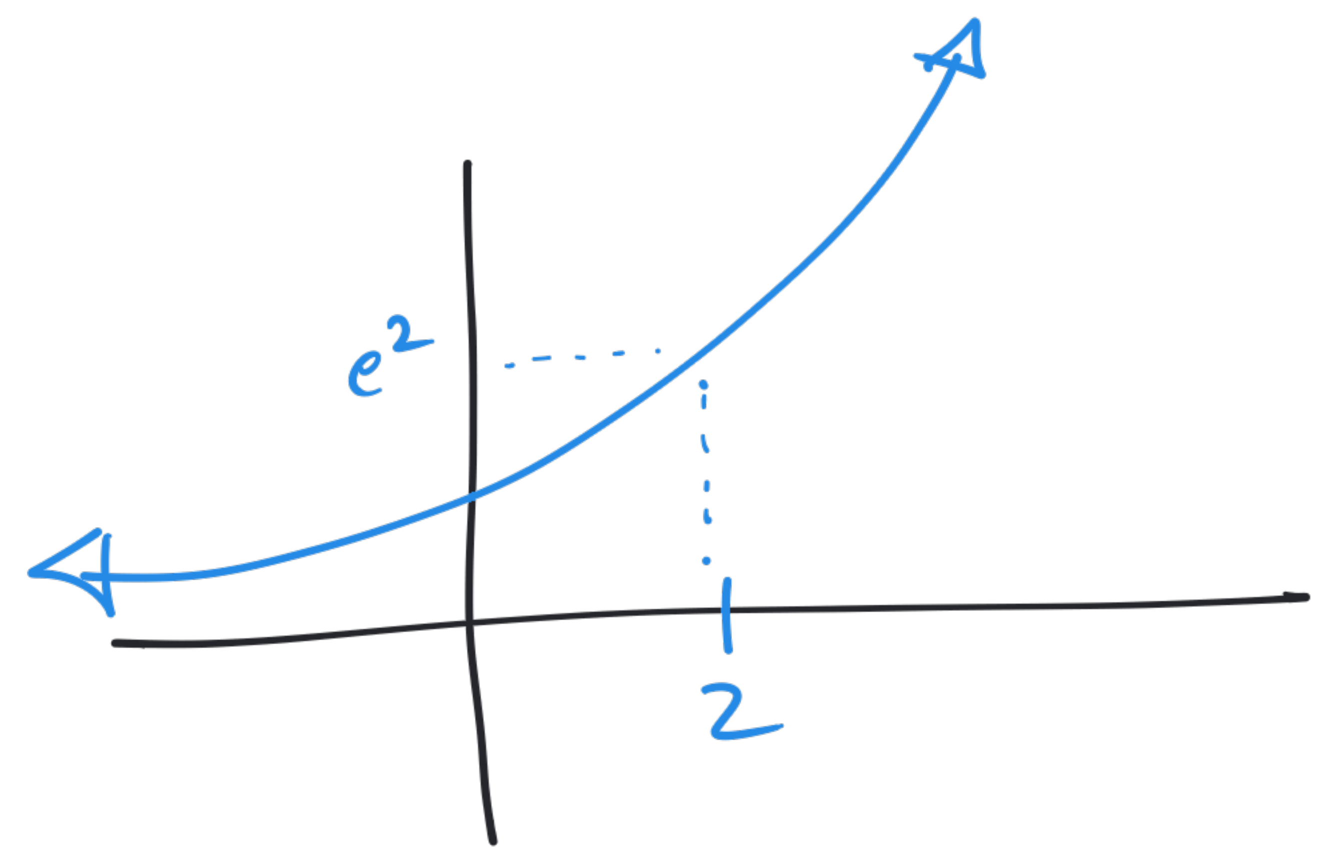
"type" $\dfrac{2e^2 - 2e^2}{2-2} = \dfrac{0}{0}$

No good

$$= \lim_{x \to 2} \frac{e^x \cancel{(x-2)}}{\cancel{x-2}}$$

$$= \lim_{x \to 2} e^x$$

$$= e^2$$



Example:

$$\lim_{x \to 3} \frac{(x-2)(x+2)}{x^2-3x-10} \leftarrow 3^2-3(3)-10 = -10$$

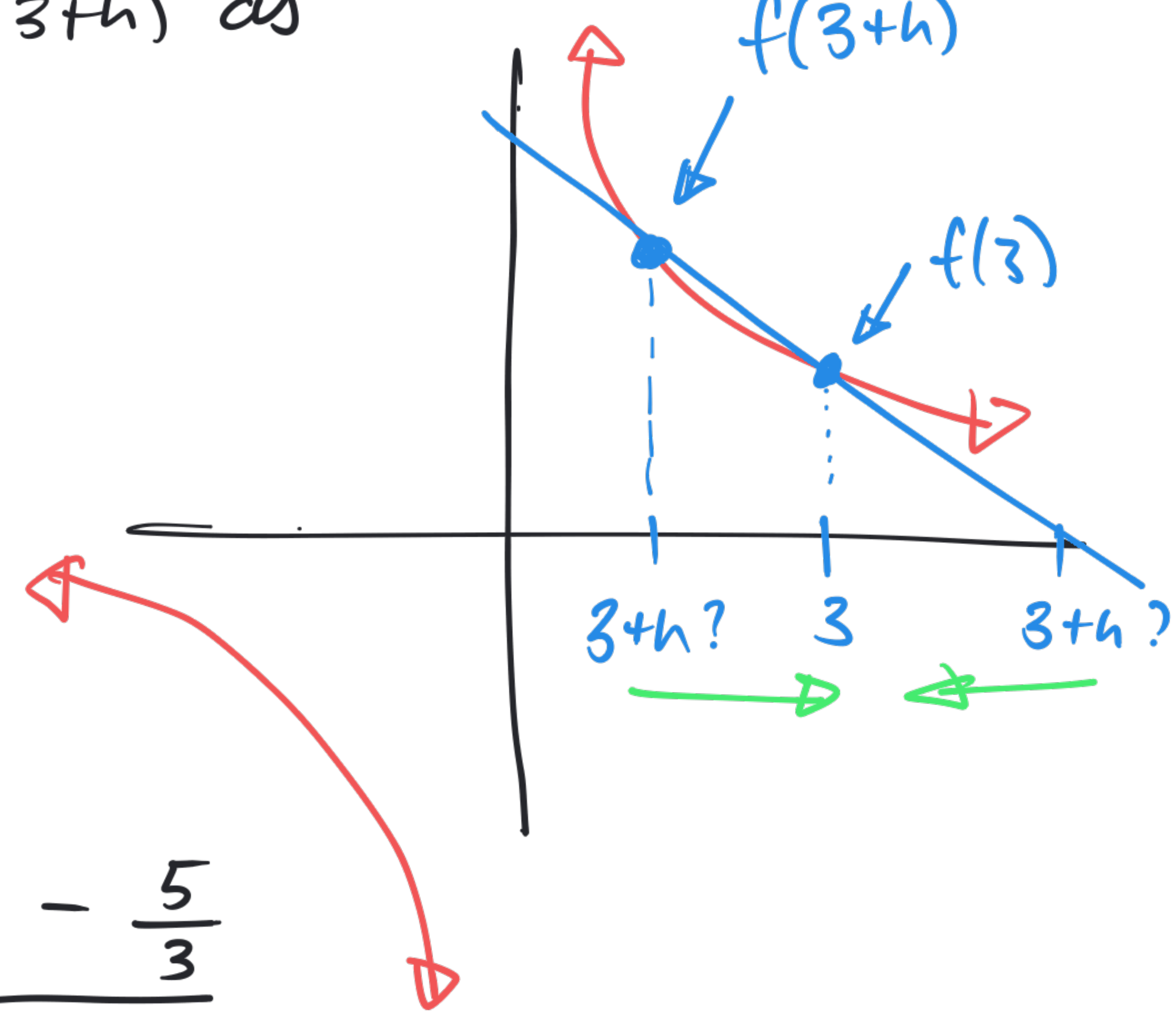
$$= \frac{\lim_{x \to 3} (x-2)(x+2)}{\lim_{x \to 3} x^2-3x-10}$$

$$= \frac{(3-2)(3+2)}{3^2-3(3)-10}$$

$$= \frac{1(5)}{-10}$$

$$= -1/2$$

Example: Let $f(x) = \frac{5}{x}$. Compute the limit of the slope of the secant line connecting $f(3)$ and $f(3+h)$ as $h \to 0$.



Slope of secant line $= \dfrac{\Delta y}{\Delta x}$

$$= \frac{f(3+h) - f(3)}{3+h-3}$$

Need to compute

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{5}{3+h} - \frac{5}{3}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{5}{3+h} \cdot \frac{3}{3} - \frac{5}{3}\frac{(3+h)}{(3+h)}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{15 - 15 - 5h}{3(3+h)}\right)$$

$$= \lim_{h \to 0} \frac{-5\cancel{h}}{\cancel{h}(3)(3+h)} = \lim_{h \to 0} \frac{-5}{3(3+h)} = \frac{-5}{3(3+0)} = \frac{-5}{9}.$$